

A Few Logical Problems of Vagueness

Daniel Jones

May 4th, 2004

Contents

1	Characterizing Vagueness	2
1.1	Context Dependence	3
1.2	Disambiguating Vagueness	4
1.3	Vagueness vs. Classical Logic	4
2	Views On Vagueness	6
2.1	Useful/Damaging	6
2.2	Inevitable/Eliminable	6
2.3	Formulable/Elusive	7
2.4	Semantic/Ontic/Epistemic	7
3	Vagueness In Vagueness	8
4	Ontic Vagueness	10
4.1	Vague Identities	11
5	Epistemic Vagueness	12
6	Semantic Vagueness	12
7	Formulating Vagueness	13
7.1	Alternative Logics	14
7.2	Supervaluations	14
7.3	Tolerance and Modeling	15
8	A Few Closing Remarks	16

“Few”. “Logical”. “Problems”. “Vagueness”. Four words that are used commonly in everyday parlance, successfully communicating nuanced ideas with little loss of understanding. Yet consider the first of the series. *Few*. It’s a fine day in late summer, and I go to my garden to pick vegetables: a few pods of peas, a few beans, a few shallots. Though I have used the same quantity term for each, nobody with a sufficient grasp of the English language would infer from this that the number of shallots and beans in my bucket would be exactly equal. Equally, nobody would happily place a precise range on the quantity of each vegetable. It’s clear that I’m not trudging home carrying three hundred shallots, yet it’s equally evident that I don’t have only one or two. Three, perhaps; seven or eight at a stretch, which would also do for a nice soup. However, it’s far from obvious that ten vegetables would qualify as a *few*. In this case, “one” and “three hundred” would clearly fall outside the bounds of the term *few*, whereas “five” falls clearly within them, at least in the author’s usage of the term. Cases such as “three” and “eight” appear questionable at best: it seems impossible to state with certainty that describing eight vegetables as “a few” would be an invalid application of the term.

1 Characterizing Vagueness

In modern philosophical usage, it is in the latter sense that we talk of the concept of vagueness; the presence of *borderline* cases in which we simply cannot say whether a concept can or cannot be applied. The topic has recently undergone a resurgence in interest, and a clamour of contradicting claims have been made as to the source and nature of this indeterminacy, which we shall examine in detail later. At present, let us take the main points of the paper that set the agenda for 20th century studies on the topic: Russell’s *Vagueness* (1923).

Briefly, Russell asserts that vagueness resides solely within the relation between representations – speech, thought, map, photograph – and what is in the world to which they correspond. He begins by discussing the concept expressed by the word “red”, saying that:

It is perfectly obvious, since colours form a continuum, that there are shades of colour concerning which we shall be in doubt whether to call them red or not, not because we are ignorant of the meaning of the word “red”, but because it is a word the extent of whose application is *essentially* doubtful

Most noteworthy is the emphasized term “essentially”. The implication is that the truth value of this predicate when related to a borderline case of redness is *intrinsically* indeterminate, and cannot possibly be ascertained under any possible extension of knowledge. Lewis (1988) offers an

attractive summary from a slightly different angle, saying that “wherever we have vague statements, we have several alternative precisifications of the vague language involved, all with equal claim to being intended” (128-129). Some vagueness can be eliminated through sufficient enquiry, referred to by Sorensen (2003) as *relative* borderline cases. Other situations may resist all attempts at clarification, in which case, Sorensen says, “there is incompleteness in the question itself”. This is an *absolute* borderline case, and it is this particular case which often causes the greatest philosophical problems.

The second characteristic that is accepted by most accounts of vagueness was also highlighted by Russell. He says that “the penumbra is itself not accurately definable, and all the vaguenesses which apply to the primary use of words apply also when we try to fix a limit to their indubitable applicability”. That is, the boundary of the borderline area is imprecise; we sometimes cannot say whether an object definitely lies within the positive extension of a predicate or its indeterminate area. Vagueness itself appears to be vague.

Later, we shall approach the phenomenon’s finer-grained details, such as whether it is a metaphysical entity of its own or a feature of language or mind, and whether there is a possibility of crafting a logical system which accounts for vagueness, along with more detailed inspections of the above characteristics. In the meantime, our current sketch will suffice.

1.1 Context Dependence

Of course, the reader may not agree with the clearly appropriate, inappropriate and borderline applications of “few” used in the introductory paragraph. This is another feature which is mooted as a characteristic of concepts whose meaning has not been precisely specified: a dependence on context, often in several regards. An age-old concept used as a prime example of vagueness is the *heap*. A heap of potatoes would in all likelihood be regarded as subject to quite different bounds of applicability than a heap of salt, thus the *subject* of the predicate brings about one context dependence. Another is imparted by the *judge* of a concept’s applicability. An ant may see fifty grains of salt as a heap, whereas a human may well not; two humans may also disagree on the heap-ness of a particular potato collection. Moreover, one particular person may judge the collection to be a heap one day and a non-heap the next. A more convincing case of this may be asking a person to state whether a turquoise towel should be described as “blue” on two occasions, the first having just emerged from a sunny, sandy beach, and the second fresh from a steamy sauna.

If vagueness is a feature of linguistic semantics, it should also be noted that an applicable range of a term may gradually alter over time. For example, a thousand years ago, a six-foot person in the Western world may have been considered particularly tall, whereas in the present day this would

be much closer to population norms.

Evidently, context affects judgement on a number of levels. This fact is not only portentous of the impending storm of problems to assail us as we attempt a systematic examination of the surrounding issues; it is also a convincing mini-demonstration of the human intellect's pervasive inconsistency. However, though ripe for studies from a psychological or purely linguistic standpoint, these aspects are of negligible importance from the philosophical stance which we are adopting, and hence shall be put aside from now on. The reader may safely assume that any general observations are intended from the perspective of a human society, proficient in English and not fresh from the beach.

1.2 Disambiguating Vagueness

The latter half of Russell (1923) discusses the difference between the relationships characterised by vague and non-vague statements. He says that “a representation is vague when the relation of the representing system to the represented system is not one-one, but one-many”. A charitable view may take this to be a primitive precursor of the recent theory of supervaluationism (see 7.2). However, most of the evidence suggests that Russell is simply conflating vagueness with what should correctly be termed *generality*. As first noted by Black (1937), vagueness is a relation between a concept and its applicable system which is intrinsically undetermined in certain cases, whereas generality is simply unspecific. General expressions can, on the whole, be narrowed down by compounding them more specific terms. Once this has been done, some cases may still display *vagueness*, whereas others may now express genuinely unique and precise names. A convincing example of a general but non-vague concept is “the prime numbers”.

Also distinct from vagueness is *ambiguity*. This is the effect of two symbols with an identical form but more than one possible relation to the world; that is, multiple possible meanings. In the case of language, all homonyms exhibit this property. Returning to the garden, the “beans” we picked may in fact be runner beans, broad beans, chick peas or some completely different variety. Vagueness may also be present in these classifications, as a vegetable taxonomist would surely testify, but this is quite a distinct property.

1.3 Vagueness vs. Classical Logic

The first signs that vagueness may prove to be a thorn in the philosopher's foot emerged in Ancient Greece, where the paradox of the Sorites was formulated. Given the enormous amount of trouble that it has thrown up, its structure is deceptively simple. It is accepted that 0 beans do not make a heap, and that adding a single bean to a given bean-collection will not transform it from a non-heap to a heap. Using the above premises, we can

repeatedly add beans to a non-heap and are eventually forced to concede that our harvest of ten thousand beans is still not a heap, despite the fact that, according to common sense, it clearly is.

The upshot of this argument is that from what are apparently true premises and a valid argument, we have derived a false conclusion – an unforgivable situation for classical logic, to which these simple rules of inference are vital.

A formulation of the argument is as follows.

- (S1) 0 beans are not a heap
- (S2) If n beans are not a heap, $n + 1$ beans are not a heap
- (S3) Therefore, n beans are not a heap

Michael Dummett (1978) has conducted a close examination of a fundamentally equivalent interpretation, Wang’s Paradox. Though this version still exploits the indeterminate bounds of a natural predicate (“small” rather than the Sorites “heap”), we are now operating within the formal, stipulated and hence intrinsically non-vague grounds of mathematical language, enabling us to be sure that the vagueness should be attributed to the predicate itself.

- (W1) 0 is small
- (W2) If n is small, $n + 1$ is small
- (W3) Therefore, every number is small

Dummett then systematically inspects each aspect of logic required for the above argument to be sound in turn. It appears that we must accept W1 and W2 according to common sense. Mathematical induction cannot be held responsible, as an equivalent formulation could be constructed from each n in turn:

- If 0 is small, 1 is small
- If 1 is small, 2 is small
- If 2 is small, 3 is small
- ...
- If $n - 1$ is small, n is small

A similar justification can be leveled at the universal quantifier required to apply (W2) to every n , and doubting the soundness of *modus ponens* or that valid chains of arguments are themselves valid appear, as Dummett says, “to violate the concept of valid argument itself” (252). According to what we’ve seen so far, classical, two-valued logic simply cannot cope with vague statements.

2 Views On Vagueness

Perspectives on the subject of vagueness vary widely. Through the past few hundred years, and through fluctuating definitions of the term itself, philosophers have speculated that it is essential and useful, a dangerous scourge that should be eliminated to the best of our abilities, and a necessary evil that we can and should incorporate into our logical thought. It has been postulated to be a feature of the world we reside in, of our thought, mind and language. I shall now proceed to briefly summarise the bodies of thought on the matter.

2.1 Useful/Damaging

Is vagueness desirable? Instinct would suggest that it isn't, and indeed "vague" is often used in a pejorative sense as if, by rights, we should have gleaming edges to all of our descriptive terms. This is the view that Gottlieb Frege took, seeing vagueness as a hateful prospect which infected our language with incoherence and a fundamental meaninglessness. Russell had a similarly negative stance, referring to a "heaven" of precise, logical terms and glumly concluding that, thanks to epidemic imprecision, even "the conceptions of truth and falsehood [...] are themselves more or less vague".

Pragmatists such as C.S. Peirce took the view, as discussed in Williamson (1994), that vagueness is inevitable and serves a useful purpose in social interaction and even within scientific study. Quine echoes a similar view, asserting that "Vagueness is a natural consequence of the basic mechanism of word learning" (Quine, 1960, 125). Steering further towards the pragmatist view, he suggests that (128):

We may prudently let vagueness persist until such pressure arises, since meanwhile we are in an inferior position for judging which reforms might make for the most useful conceptual scheme.

On the other side of the coin, Crispin Wright's account is that "the utility and point of the classifications expressed by many vague predicates would be frustrated if we supplied them with sharp boundaries" (1975, 330).

2.2 Inevitable/Eliminable

Frege and Russell are similarly allied in their belief that although we *should* eliminate vagueness, it seems to inescapably invade the language with which we talk about the world. As Williamson vividly describes the situation, "A citadel of precision is needed from which to stipulate outwards" (1994, 169).

The generally accepted view was put forward by Black (1937). With characteristic eloquence, he asserts that (27):

...deviations from the logical or mathematical standards of precision are all pervasive in symbolism; [and] to label them as subjective aberrations sets an impassable gulf between formal laws and experience and leaves the *usefulness* of the formal sciences an insoluble mystery. (*author's own emphasis*)

2.3 Formulable/Elusive

Black's paper proceeded to devise a schema for linguistic vagueness based on a statistical analysis of a sampled usage of terms, gauging the vagueness of a concept by the spread of its "consistency profile". Though this approach has now generally been set aside by the philosophical fraternity, it set the agenda for numerous attempts to provide a logical systematization. Work such as Dummett's (1978) has convincingly demonstrated that the chances of doing so using classical, bivalent logic in its vanilla form are practically zero, and so research has shifted to multiple valued logics, utilising neutral truth values or degrees of truth (which we shall see in section 7.1) and theories which veil parts of logic from us altogether (section 7.2).

Others suggest that a fully explanatory account of vagueness in a formal language is simply inconceivable. Ludwig Wittgenstein's view was that linguistic vagueness is so important that any complete account would require a level of complexity equal to that of the language it is describing (discussed further in Williamson 1994, 84-89). An acceptable compromise may be to approximately model vagueness to a *sufficiently useful* degree – a formalization in the spirit of the pragmatists.

2.4 Semantic/Ontic/Epistemic

The greatest debate in contemporary philosophy of vagueness is over where exactly its roots lie. It is fairly evident where it manifests itself, language providing the most salient example. Does this indicate that there was no vagueness before the time of the human mind? Presumably not, as we must surely assume that animals are able to form reasonably consistent yet flexible concepts to be able to, for example, identify and avoid their predators.

Yet what are these concepts built on? The world, or, as some may have it, the *facts* which indicate the world's state of affairs. If we posit that our concepts are precise and the facts that are vague, vagueness is an artefact not just of our representations but of the world itself. Another case for this may be Sorensen's (2003) example of "seeing stars" – an indeterminate number of objects whose vagueness cannot be straightforwardly pinned down to our representations. Moreover, indeterminate statements of identity have recently offered a renewed motivation for ontological vagueness following a brief, controversial article by Evans (1978).

A third view is that backed by Williamson (1994): the *epistemic* conception of vagueness. This claims that vague terms do in fact have precise boundaries, but we are (inevitably) ignorant of precisely where they lie.

Wherever vagueness does actually reside, examples such as Wang's Paradox unambiguously demonstrate that it afflicts predicates especially palpably. For this case, and following Fine (1975) in the belief that all (semantic) vagueness can be restated as predicate vagueness, most examples from hereon will put prime consideration to predicates.

3 Vagueness In Vagueness

The term "vague" is, as Hyde (1994) points out, homological. It is a familiar fact to anybody to have grappled with the topic that attempting to pin down the boundaries of a vague concept is akin to catching an eel with bare hands. Why should this be the case?

Between collection of beans that clearly qualifies as a heap and one that is clearly a non-heap, there are others of indeterminate heapness. However, just as it seems counterintuitive to suggest that adding one bean to a heap can turn it into a non-heap (S2 of the Sorites Paradox), it seems equally as absurd to claim that adding one bean transforms the collection from being definitely not a heap to a candidate of unknown applicability. Thus, the borderline cases have borderline cases of their own, and borderline borderline cases have borderline cases, and so on, *ad infinitum*. This is known as *higher order vagueness*.

A particularly ingenious proof of the necessity of higher order logic was given by Sorensen (1985). He posits a series of predicates, *n*-small, indexed by the natural numbers, which apply to those integers either small or less than *n*. These are then shown to be Sorites-susceptible for the predicate "vague":

- (NS1) 1-small is vague
- (NS2) If *n*-small is vague, then (*n* + 1)-small is vague
- (NS3) Therefore, *n*-small is vague

Now take *n* to be 10^6 , for example. 10^6 -vague clearly applies to all numbers less than 10^6 , and clearly does not apply to any equal or greater. Thus, Sorensen claims, "vague" is vague.

The argument seems sound, and it also suits the intuitive perspective that vagueness itself has indeterminate bounds. Deas (1989) offers a counterargument based on the premise that the vagueness of the *n*-small series piggybacks on the vagueness of "small". He notes that the ability to even be able to construct a paradox such as this relies on the presence of vague predicates other than "vague" itself. We shall ignore the question of whether

this scenario is even possible, but instead simply note that the n -small series simply *demonstrates* the potential for indeterminacy of “vague”. As Varzi (2003) notes, we can create complex predicates whose concepts genuinely are vague, regardless of whether their own indeterminacy relies on vague constituents. In this respect, Sorensen’s argument is valid.

Hyde (1994) believes that this higher order vagueness is an essential property of vagueness as a whole. He takes Sorensen’s proof along with the intuitive argument that, if every constituent of a compound is precise, the compound is also precise. Finally, we are given a pared-down definition of vagueness as “the existence of a border case”. Since the only ambiguous constituent of this is “border case”, we should accept that this too is vague.

Hyde’s paper has a number of minor fallacies lurking under the surface, though most have little bearing on its ultimate goal. In the introduction, the predicate “child*” is cited by as an example of a predicate which does *not* exhibit true vagueness. This term is defined as applicable to any person below sixteen years old, inapplicable to anybody over eighteen, and indeterminate for anybody between these two ages. That is:

- (C1) n is a child* if $age(n) < 16$
- (C2) n is not a child* if $age(n) > 18$
- (C3) n is indeterminately a child* if $16 \leq age(n) \leq 18$

Hyde rightly describes this as a mistaken candidate for a predicate that is vague *in the way that the example is intended*. However, somewhat ironically, he seems to overlook the pervasiveness of vague qualities. If we accept that $age(n)$ is itself a precise term, the proof is entirely correct. Yet as Russell took care to point out, the process of being born is subject to uncertain boundaries, as is the measurement of seconds and years – and accepting this entails that the C1/C2 and C2/C3 boundaries are each vague. Thus, the above *is* in fact a vague concept, albeit one with a rather strange profile of two very narrow penumbra.

Compare this to the example given by Fine (1975):

- (N1) n is nice₁ if $n > 15$
- (N2) n is not nice₁ if $n < 13$

The effect that Fine intended was to demonstrate the predicate’s underdetermination. There are clearly cases in which its applicability is unspecified. However, this is distinct from Hyde’s case in that the case of $16 \leq age(n) \leq 18$ has not been stipulated at all. Eklund (2003) raises the charge that Fine’s example “effects a sharp and scrutable tripartite division” (11) yet this is simply not the case.

As an analogy – though one which should be considered only as an approximate model – consider a spectrum of colour, ranging continuously

from red to orange and marked with natural numbers spread evenly from 0 to 28, with 0 corresponding to red and 28 to orange. If we are able to assert that a point n is a definite positive candidate for “orange”, it follows that, since any points numbered higher than this are bound to be orange to a greater degree than is n . The converse follows for “red”; any point numbered less than a definitely red point n must also be (definitely) red. Now suppose that 15 is a clear positive candidate for “orange” and 13 for “red”. This a paradigmatically vague situation, and – as the reader will surely have noticed – identical to the system that Fine has composed above, save for the phenomenological complications.

Later, Hyde (1994) offers an argument for the necessity of defining “vague” using vague terms which also seems somewhat flimsy. He comments that (40):

...we are not characterising “vague” using the term “vague”; rather we are characterising “vague” using vague terms and this is no more a problem than characterising “meaningful” in meaningful terms. In fact, far from being viciously circular, it is required of us that we characterise “vague” in vague terms for exactly the same reason that we are required to characterise “meaningful” in meaningful terms.

Although we may not be able to precisely define the meaning of either term, it is a fallacy to assume that both *must* be described using themselves simply because they are homological. “Meaningful” must be characterised meaningfully because this is analytic of any definition; a definition must be meaningful or it cannot serve to define. “Short”, conversely, is homological (accepting the disambiguation of shortness in terms of word length) but its shortness isn’t one of its essential features.

However, it still seems unavoidable to concede that higher order vagueness *is* an inevitable product of any vagueness at all. Indeed, this conforms with our intuitive expectations so it is not a situation to be balked at based the complication that it apparently bestows.

4 Ontic Vagueness

Russell (1923) held that attributing vagueness to anything that is not a representation was a category error, an example of what he called “verbalism”: “the fallacy that consists in mistaking the properties of words for the properties of things”. Things in the world are exactly what they are, nothing more, nothing less, and this seems a watertight proposition.

We’re out once more in the garden, and look up to notice a single cloud in the sky, appearing as a single, distinct object. If we zoomed in to a few centimetres away, we would see that it is composed of billions of vapour

particles with no clear bounds; it is not clear where the cloud begins or ends, nor can we tell when we're definitely inside it or definitely within its border. The analogy of the cloud's fuzziness to that of a vague predicate is clear to see, and it is evident why some philosophers would claim a cloud to be a vague object. However, this view makes the assumption that a cloud, an entity residing in the empirical world, is a single "object" as such, when object-ness is a concept created by our minds. Russell is right, of course; empirical things are what they are, and nothing more.

4.1 Vague Identities

So, a chair is what it is. What if we chip a piece of wood from it? Does it retain the same *identity*? If so, we can chip another piece of wood off, and its identity will presumably stay intact. Acting as a slow motion wood chipper, we eventually turn the chair into a mulch for our beloved garden, with the assumption either that the chair is still the same object or that one particular chip was the one that broke its metaphorical back. This ontic Sorites poses a question which has recently sparked a great deal of debate: can real objects be indeterminately identical? If so, it may be inferred, vagueness *is* an ontological artefact, of objects or at least of the properties they exhibit.

The debate revolves around the aligned theories of Evans (1978) and Salmon (1982), both of which purport to prove that (and, strangely, both of which are virtually footnotes – the former being a one-page article and the latter an appendix entry). Though an epistemicist may take umbrage at the strength Salmon's comment that "there is no *objective* fact of the matter" (**emphasis added**), the general message of his and Evans' work is frustratingly compelling. By assuming that two objects x and y are *indeterminately* identical, we can show that x has a property that y does not have – namely, being determinately identical to x . Thus, using Leibniz' law that identicals must possess all of the same properties, we have shown that x is determinately non-identical to y .

This is not to say that no identity statements can be vague, as this can easily be shown not to be the case by two indeterminate names. What it *does* show is no that identity statements can be vague between determinately named objects which are vague *in themselves*.

One of the better papers to emerge from the furore of interpretations is that of Cook (1986), who insists that the final step in Evans' argument fails because the property which n and y do not have in common is not an "ordinary property" as it is defined in terms of indeterminacy. This done, Cook then goes on to form a cogent disproof of indeterminate identity based upon similar premises to the theory behind supervaluationism, which shall be discussed later.

As Williamson (1994) comments, identity statements are just one po-

tential facet of the world that could exhibit vagueness, and a disproof of vague identities still leaves room for other potential ontic indeterminacies. However, through all of the above, there remains a faint odour of misguidedness in the air, applying mental and metaphysical arguments to an empirical world. Even if there is no determinate fact whether a given object exemplifies a given property, the characterisation of the property relies on us and hence without our conceptual constructs, the *possibility* of the fact would dissipate.

As far as I can see, no satisfactory account has yet been composed of true ontic vagueness, and so we shall assume that vagueness supervenes upon precise ontic facts.

5 Epistemic Vagueness

Williamson (1994) is a supporter of ignorance. The epistemic conception of vagueness states that the concepts we perceive to be vague do in fact have precise and objective boundaries, but boundaries that are intrinsically unknowable. The inconsistency in our usage is due to this ignorance. This approach seems initially unintuitive, as it has hints of language being governed by something above human knowledge when it is clearly a human social construct. On the other hand, if we take a stance similar to that behind Black's statistical analysis and conduct a general survey of a term's usage, we would find a mean point around which individual trends are distributed.

However, not only is this impractical but it is misleading. As described in "Context Dependence", the applicability of a concept depends on such a wide range of factors that the expected mean at a given point is impossible to gauge – and it is this fact which ensures that our ignorance will never be overcome. Interestingly, the epistemic conception fits happily on top of classical logic, cooperating with the tricky principle of bivalent truth; indeed, Williamson apparently proves by RAA that bivalence is *necessary* within asserted borderline cases.

6 Semantic Vagueness

The most obvious and pervasive manifestation of vagueness is in the language we use from day to day. We converse using terms whose bounds we could not pinpoint precisely, and neither could our conversational partners, yet our usage overlaps and is regular to a sufficient degree to allow a virtually perfect rate of success in communication. So how and why does vagueness come about?

Frege saw natural language as built up on a framework of atomic terms, each of which *refers* to an object in the world. A name refers to an object,

and a predicate (applied to a particular subject) refers to a truth value. An expression whose application is undefined in some cases is *incomplete* – and in virtue of this, has no referent, meaning that it is not a valid concept at all. According to Frege, our accommodations for vagueness inevitably lead to incoherence.

“Vagueness is a precondition of the flexibility of ordinary language”, says Williamson (1994, 70). Most linguists would agree, and this seems intuitively correct. Strengthening this position, it seems likely that vagueness is not just useful but inevitable, given the language acquisition mechanisms that we must employ. Assuming the typical example of redness, Quine (1960) discusses *ostensive* learning, wherein a child picks up terms by demonstration and reward. As the concept coagulates and sharpens in the child’s mind, stimuli to which it applies form a “distribution about a central norm” (85; *emphasis suppressed*) in a sensory “quality space”. Though the theories of reward-learning and the social function of language lie outside the scope of this paper, what is important is that concepts are developed demonstratively, with no stipulation being made as to where precisely the red-orange border lies.

These *observational* predicates, however, throw another spanner in the works. The world around us is continuous whereas our visual perception is discrete; we can only perceive a finite number of distinct areas. This will be returned to briefly when we examine tolerance (section 7.3).

7 Formulating Vagueness

As we have seen, the epistemic account of vagueness attempts to map our perspective of the world onto the framework set by classical logic. Whether this is valid or not has been left unresolved, though there are clearly some common sense barriers to accepting the thesis.

An alternative method is to attempt to remould classical logic into a novel system which could take issues of indeterminacy in its stride, ensuring that the qualities vital to any logical system are left in place. Enumerating a few of these logical principles, we have:

Truth-functionality - the truth value of a complex expression should be derivable from its atomic sub-expressions.

Valid Reasoning - we should be able to progress from theorem to theorem using *modus ponens* or an equivalent inference relation and using the set of designated values as criteria for successful inference.

In addition, there are a number of features that we should ideally demand of any formal theory of *vagueness*:

Vagueness-functionality - a non-contradictory phrase containing any vague atoms should also be vague, whereas a phrase containing only precise atoms should be entirely precise.

Penumbral Connections - account should be taken of interconnected terms within a borderline case, in particular allowing for the law of excluded middle.

7.1 Alternative Logics

Accepting the premises that vagueness entails unavoidably indeterminate statements, it is a natural step to attempt to formalize this requirement in a logical language which caters directly for such indeterminacies. Two options immediately offer themselves; many-valued logics (normally containing T, F and a neutral term representing indeterminacy) and degrees of truth.

The central problem with many-valued logics is that important rules of classical logic are invariably lost along the way, no matter how the system is defined. Williamson (1994) demonstrates cases in which serious damage is done to the law of excluded middle, the law of non-contradiction, the loss of generalized truth-functionality and even a mutilated *modus ponens*. Similar issues appear to occur with many logics of degree. Moreover, Williamson suggests, higher order vagueness is lost as we assign precise numerical truth degrees to a statement, though we shall see a satisfying and straightforward approach to this in 7.3.

7.2 Supervaluations

The supervaluationist theory of vagueness is built upon the idea that vague concepts are underdetermined; statements applying these concepts to borderline objects have no truth value. However, a vague concept now represents the entire range of its possible precisifications over a given object; each has its own true/false territory, and the concept as a whole now has a *supertruth*. A proposition is supertrue if it is true under every precisification which conforms with the predicate's admissible criteria.

Supervaluation theory's major advantage over, for example, fuzzy logic is that it is able to take into account *penumbral connections*. When p and q are indeterminate, $p \vee q$ should also be indeterminate. However, in the case of $p \vee \neg p$, the law of excluded middle should rightfully hold. In supervaluationism, it does, which is a vital advantage in cases such as the problems of indeterminate identity cited in Cook (1986).

However, it is scarcely free from problems. Williamson (1994) shows that supervaluationism is incompatible with the Tarskian disquotationalism accepted by many theories of truth. More importantly, we lose vital reasoning tools such as \vee -elimination and *reductio ad absurdum*, which are enormous dents in any logical system which lays claim to being relatively complete.

7.3 Tolerance and Modeling

The belief attributed to Quine – namely, that most empirical predicates are developed ostensively, and it is this which is the root of our semantic vagueness – is backed on the whole by Wright (1975), who proposes a *governing view* of language use. This has two main principles:

- (G1) The application of a particular linguistic expression is determined by rule.
- (G2) Properties of the rules governing language can be determined by considering, in the case of each application, its justification, experiential variables and potential consequences.

Wright next highlights and accepts an incoherence between these two principles. Since our intellect is fundamentally inconsistent, and we must inevitably take into account conflicting situational variables, applying G2 to each concept judgement often results in us being forced to break G1. The upshot of this is that the cutoff points of concepts develop the blurred boundaries of higher order vagueness. This apparent incoherence is not a problem for Wright, who accepts that vague predicates would often lose their usefulness were they to be sharpened.

Wright goes on to point out that, in a series of Sorites-like paradoxes of degree, a ubiquitous feature is “a certain *tolerance* in the concepts respectively involved, a notion of a degree of change too small to make any difference” (333). He posits a concept θ which is represented by the predicate F , and we can add a particular object α , which is subsumed under θ – that is, $F(\alpha)$ is true. If α now alters in degree in terms of its θ -ness yet $F(\alpha)$ remains absolutely true, θ is a vague concept.

The key feature of this tolerance, and one which is characteristic of *all* formulations of the Sorites paradox, is the *intransitivity of indiscriminability* of the concept involved. Let us take the example of a continuum of colour ranging from red to orange, and select an arbitrary series of points $p_0 \dots p_n$, where p_0 is indiscriminable from p_1 , p_1 is indiscriminable from p_2 and so on.

This property has the most striking implications when we consider phenomenological continua – that is, Sorites-susceptible fields over observational concepts. This is discussed in more detail in Wright (1975).

Some theorists deny outright that a formal equivalent of vagueness can be constructed. Wittgenstein effectively claimed that, in Tarskian terminology, the metalanguage used would have to be at least as rich as the object language to fully capture the dizzying subtleties of natural vagueness. Some more modern examples of similar scepticism are discussed in Cook (2002): R.M. Sainsbury demonstrates that a set-theoretic approach, fuzzy or nay, has an intrinsic determinacy which will emerge at some level. This directly contrasts our view of vagueness, which has a limitless *indeterminacy*.

The solution which Cook (2002) proposes is astonishingly obvious yet possibly the most intuitively appealing proposition so far. Rather than treat a logical formulation of vagueness as a homeomorphic description of empirically occurrences of vagueness, we should treat it as a *model* of vagueness. A model, in Cook's view, is differentiated from a proper description by virtue of being *rough*. This embraces our standing definitions of vagueness and generality (in the sense that a slightly blurry photograph is "general") to a sufficiently low degree to still be useful, plus a novel property of containing potential *artefacts*. These artefacts are attributes of the model which not represent any actual features of the system being modeled, but "are not necessarily useless or eliminable [...] since they might be crucial to the structural integrity of the model".

8 A Few Closing Remarks

In the current state of affairs, we have no one logical formulation which provides an entirely appropriate description of vagueness as it is intuitively viewed. Moreover, it appears that, since formal language is locked within its citadel of precision, no such model could be satisfactorily produced. However, using the vague modeling proposed by Cook (2002), we can focus on the rough *positive* resemblances offered by models such as supervaluations and fuzzy logic, and apply them wherever appropriate. Though this has the air of a non-solution to a problem which appears to desire a logical outcome, I believe that no purely precise system can be applied with full descriptive powers to one that is vague.

References

- Black, M. (1937, October). Vagueness: An exercise in logical analysis. *Philosophy of Science* 4(4), 427–455.
- Cook, M. (1986). Indeterminacy of identity. *Analysis* 46, 179–186.
- Cook, R. T. (2002). Vagueness and mathematical precision. *Mind* 111, 225–248.
- Deas, R. (1989). Sorensen's sorites. *analysis* 49, 26–31.
- Dummett, M. (1978). *Truth and Other Enigmas*. Cambridge, MA: Harvard University Press.
- Eklund, M. (2003). What vagueness consists in (in progress). (<http://spot.colorado.edu/~eklundm/wvci.pdf>) [accessed 4 May 2004].
- Evans, G. (1978). Can there be vague objects? *Analysis* 38, 208.

- Fine, K. (1975). Vagueness, truth and logic. *Synthese* 30, 265–300.
- Hyde, D. (1994). Why higher-order vagueness is a pseudo-problem. *Mind* 103, 35–41.
- James, E., J. Slater, et al. (Eds.) (1983). *The Collected Papers of Bertrand Russell*. London: Allen & Unwin/Unwin Hyman.
- Lewis, D. (1988). Vague identity: Lewis misunderstood. *Analysis* 48, 128–130.
- Quine, W. (1960). *Word and Object*. Cambridge: MIT Press.
- Russell, B. (1923). Vagueness. *Australasian Journal of Philosophy and Psychology* 1, 84–92. Reprinted in James et al. (1983).
- Salmon, N. U. (1982). *Reference and Essence*. Oxford: Basil Blackwell.
- Sorensen, R. (2003). Vagueness. In *The Stanford Encyclopedia of Philosophy*. (<http://plato.stanford.edu/archives/fall2003/entries/vagueness/>) [accessed 4 May 2004].
- Sorensen, R. A. (1985, June). An argument for the vagueness of “vague”. *Analysis* 45(3), 134–137.
- Varzi, A. C. (2003). Higher-order vagueness and the vagueness of ‘vague’. *Mind* 112, 295–298.
- Williamson, T. (1994). *Vagueness*. London: Routledge.
- Wright, C. (1975). On the coherence of vague predicates. *Synthese* 30, 325–365.